INFORMATION ENGINEERING IN SIGNAL AND IMAGE PROCESSING

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Abstract

Ever-expanding general discipline of signal and image processing occupies a very important position in information engineering forming a basis for further disciplines including control engineering, vision, robotics, biomedical image analysis and environmental signal processing. The paper presents the integration role of this interdisciplinary area connecting physics and mathematics and it provides the survey of selected methods of time-frequency and time-scale analysis using the short-time Fourier Transform (STFT) and wavelet Transform (WT) at first. Comparison of both methods imply the different and varying resolution in case of the wavelet transform and it presents some their properties. Signal decomposition enables very important applications in signal de-noising, compression, segmentation and classification. These general methods can be applied in many areas including biomedical image analysis and environmental signal processing. A special attention is paid to the use of signal processing in signal prediction.

1. INTRODUCTION

Signal and image processing became an integral part of many engineering disciplines in the last century allowing to find similar mathematical description of diverse applications including biomedical image analysis, environmental signal processing, control system modelling, speech analysis and data forecasting. In this way it forms an interdisciplinary basis for physics and mathematics using information engineering and modern information technologies.

Methods and applications of *signal and image processing* as an ever-expanding discipline are discussed in many papers and books [33,14,2,31,7,5,25,17,21]. In some way it reminds the relation between mathematics and physics expressed by famous French mathematician Henri Poincaré: *The science of physics does not only give us (mathematicians) an opportunity to solve problems, but helps us also to discover the means of solving them*. In similar way Simon Haykin, professor of McMaster University in Canada says [9]: *Signal processing is at its best when it successfully combines the unique ability of mathematics to generalize with both the insight and prior information gained from the underlying physics of the problem at hand*.

Information engineering assume digital real data acquisition with a selected sampling period and their following analysis and processing including system identification and linear or non-linear modelling. In this way information engineering and signal and image processing provide a general tool for control engineering, measuring engineering, image processing, vision, robotics and other related discipline using database engineering in many cases. It allows both system and signal modelling allowing signal segmentation, feature extraction, classification, prediction or compression.

Mathematical methods of signal and image analysis are based in many cases on the one-dimensional or two-dimensional discrete Fourier transform or on the wavelet transform [18,11,35] allowing either time-frequency or time-scale signal analysis. The following signal and image processing use both linear methods including FIR filters described in z-domain and non-linear methods based upon artificial neural networks [8,1] using various optimization methods in many cases. Further application allows signal components rejection providing a basis for adaptive signal processing and signal and image enhancement and de-noising [22,26].

Algorithmic tools using information technologies allow verification and application of general methods of signal processing. The paper is devoted to the survey of selected methods of time-frequency and time-scale signal analysis used for signal components detection and classification, to selected topics of linear and non-linear signal modelling allowing its prediction and to basic comments to image processing. Applications include biomedical signals [15], analysis of energy consumption [29,30], environmental signals [12] and image processing [20,19,13]. The mathematical description and analysis of such systems is given in the MATLAB/SIMULINK environment [10] resulting in algorithms verified for simulated signals at first and applied to real signal processing.

2. SIGNAL ANALYSIS

Information engineering uses methods of digital signal processing for time-frequency and time-scale signal analysis forming fundamental tool in signal decomposition, feature extraction, classification and processing including signal de-noising.

1. Discrete Fourier Transform

In case of an analysis of a signal $\{x(n)\}_{n=0}^{N-1}$ it is possible to apply the Discrete Fourier Transform and namely its fast algorithm (FFT) that enables to find its transform in the form

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\,k\,n\,2\pi/N} \tag{1}$$

for $\omega(k) = k\frac{2\pi}{N}$ where $k = 0, 1, \cdots, N-1$. This transform can be applied either to a selected signal segment or it can be applied in a moving window as a short-time Fourier transform (STFT) allowing non-stationary signal analysis. Frequency and time resolution is constant and longer window results in worse time resolution and vice versa.

Fig. 1 presents the use of STFT for the analysis of energy (gas) consumption measured with the sampling period of two hours. The time representation of this signal enables to assume the periodic component one day long that is verified by the FFT that detects periodicity of one day presenting higher spectral components as well. This result can be further used in this time series modelling, model order selection and its prediction.

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Figure 1. The short time Fourier transform of a selected segment representing gas consumption in the Czech Republic measured with the sampling period of two hours

2. Two-Dimensional Fourier Transform

Methods of image processing are closely related to two-dimensional discrete Fourier transform of signal $\{g(m,n)\}_{n=0}^{N-1}\,_{m=0}^{M-1}$ defined by relation

$$G(k,l)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g(m,n)\, e^{-j\, ln\, 2\pi/N} e^{-j\, km\, 2\pi/M}$$

for $k=0,1,\cdots,M-1$, $l=0,1,\cdots,N-1$ standing for frequency components

$$F_1(k) = k\; 2\pi/M , F_2(l) = l\; 2\pi/N$$

Results of simulated image analysis are presented on Fig. 2.

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Figure 2. Simulated image analysis based upon the application of two dimensional Fourier transform and its added noise rejection by two dimensional FIR filter

3. Discrete Wavelet Transforms

Wavelet transforms (WT) provide the alternative to the short-time Fourier transform for non-stationary signal analysis [27,34,3,18]. Both STFT and WT result in signal decomposition into two-dimensional function of time and frequency respectively scale. The basic difference between these two transforms is in the construction of the window function which has a constant length in case of the STFT (including rectangular, Blackman and other window functions) while in case of the WT wide windows are applied for low frequencies and short windows for high frequencies to ensure constant time-frequency resolution. Local and global signal analysis can be combined in this way.

Wavelet functions used for signal analysis are derived from the initial basic (mother) function $h(t)$ forming the set of functions

$$h_{m,k}(t) = \frac{1}{\sqrt{a}}\, h\,(\frac{1}{a}\,(t-b)) = \frac{1}{\sqrt{2^m}}\, h\,(2^{-m}\, t - k)) \qquad (2)$$

for discrete parameters of dilation $a = 2^m$ and translation $b = k\; 2^m$. Wavelet dilation correspond to spectrum compression according to Fig. 3. The most common choice includes Daubechies wavelets even though their frequency characteristics stands for approximation of band-pass filters only. On the other hand harmonic wavelets introduced in [18] can have broader application in many engineering problems owing to their very attractive spectral properties.

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Figure 3. Spectral analysis of selected wavelet functions presenting relation between time dilation and the corresponding spectrum compression

The basic efficient way to evaluate wavelet transform coefficients using the signal processing notation assumes implementation of the Mallat's pyramidal structure of wavelet transform coefficients evaluation for a given column vector $\{x(n)\}_{n=0}^{N-1}$ according to the scheme given in Fig. 4.

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Figure 4. A pyramidal filter bank structure used to evaluate wavelet transform coefficients for a given signal $\{x(n)\}_{n=0}^{N-1}$ and complementary low-pass and high-pass filters $l(n)$ and $h(n)$

Using the signal processing point of view this algorithm assumes the use of the half band low-pass scaling sequence $\{l(n)\}_{n=0}^{L-1}$ together with the corresponding wavelet sequence $\{h(n)\}_{n=0}^{L-1}$ and their convolution with the analyzed signal $\{x(n)\}_{n=0}^{N-1}$ subsampled by two using relation

$$p(n)=\sum_{k=0}^{L-1} l(k)\, x(n-k) = \sum_{j=n,n-1,...}^{n-L+1} x(j)\, l(n-j)$$

$$q(n)=\sum_{k=0}^{L-1} h(k)\, x(n-k) = \sum_{j=n,n-1,...}^{n-L+1} x(j)\, h(n-j)$$

Matrix notation given in Fig. 4 assumes convolution matrices

$$\mathbf{L}=\begin{bmatrix} l(1) & l(0) & 0 & 0 & \cdots \\ l(3) & l(2) & l(1) & l(0) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & l(L-1) & \cdots & l(0) \end{bmatrix}$$

$$\mathbf{H}=\begin{bmatrix} h(1) & h(0) & 0 & 0 & \cdots \\ h(3) & h(2) & h(1) & h(0) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & h(L-1) & \cdots & h(0) \end{bmatrix}$$

allowing signal decomposition into the following form

$$x(n) = a_0 + \sum_{m=0}^{s-1} \sum_{k=0}^{N/2^{m+1}-1} a_{2^m+k}\, h(2^{-m}\, n-k) \tag{3}$$

Comparison of real EEG signal analysis both by the STFT and WT is given in Fig. 5. Peak detection is much more accurate by the WT comparing with the STFT owing to its changing window size.

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Figure 5. Spectrogram and scalogram of a given data segment representing EEG signal

3. SIGNAL AND IMAGE PROCESSING

Information about signals resulting from a selected process can be based upon signal decomposition by a given set of wavelet functions into separate levels or scales resulting in the set of wavelet transform coefficients. These values can be used for signal compression, signal analysis, segmentation and in case that these coefficients are not modified they allow the following perfect signal reconstruction. In case that only selected levels of signal decomposition are used or wavelet transform coefficients are processed it is possible to extract signal components or to reject its undesirable parts.

1. Signal and Image De-Noising

Using the threshold method introduced by [28,6] it is further possible to reject noise and to enlarge signal to noise ratio. The de-noising algorithm assumes that the signal has low frequency components and that it is corrupted by the additive Gaussian white noise with its power much lower than that of the analyzed signal. The whole method consists of the following steps:

- Signal decomposition using a chosen wavelet function up to the selected level and evaluation of wavelet transform coefficients
- The choice of threshold limits for each decomposition level and modification of its coefficients
- Signal reconstruction from modified wavelet transform coefficients

Results of this process depend upon the proper choice of wavelet functions, selection of threshold limits and their use.
The application of threshold limits to modify wavelet coefficients $\{c(k)\}_{k=0}^{N-1}$ include two basic approaches. The use of the soft thresholding formula for a chosen thresholding value δ results in the evaluation of new coefficients

$$\bar{c}_s(k) = \begin{cases} \text{sign } c(k)\ (�c(k)� - \delta) & \text{if } �c(k)� > \delta (4) \\ 0 & \text{if } �c(k)� \leq \delta \end{cases} \quad (5)$$

The hard thresholding method results in the following values of coefficients

$$\bar{c}_h(k) = \begin{cases} \text{sign } c(k) & \text{if } �c(k)� > \delta (6) \\ 0 & \text{if } �c(k)� \leq \delta \end{cases} \quad (7)$$

Similar approach can be applied both for one-dimensional and two-dimensional signals. Further methods of image de-noising include the two dimensional convolution of the kernel $\mathbf{H}_{K,J}$ and the image data $\mathbf{A}_{M,N}$,

$$-6mmB(m+K/2, n+J/2) = \sum_{k=1}^{K} \sum_{j=1}^{J} h_{k,j}\ A_{m-k,n-j}$$

Results of application of this method for biomedical image processing and enhancement is given in Fig. ??0.

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Figure 6. Magnetic resonance image de-noising and enhancement

2. Image Interpolation and Correlation

Interpolation belongs to fundamental methods in many applications. A special role has two-dimensional or three-dimensional interpolation using either linear, cubic or spline methods. A specific application is in interpolation of air-pollution measured at specified observation stations defined by their latitude and longitude to the selected region.
For information about air pollutants it is possible to use correlation methods applied to images obtained from satellites at different wavelengths. Correlation coefficient for corresponding subimage regions of matrices $\mathbf{A}$ and $\mathbf{B}$ can be evaluated by relation

$$-6mmR = \frac{\sum_m \sum_n (A_{m,n} - \bar{A})(B_{m,n} - \bar{B})}{\sqrt{(\sum_m \sum_n (A_{m,n} - \bar{A})^2)(\sum_m \sum_n (B_{m,n} - \bar{B})^2)}}$$

Selected results of air pollution obtained from correlation between two satellite channels are presented in Fig. 7.

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Figure 7. Air pollution by dust particles obtained from correlation between two channels of satellite observations

3. Signal Modelling and Prediction

Signal modelling is a very important tool of information engineering with its application in technology, control systems, bioengineering, environmental systems and econometrics. Methods of signal modelling and prediction include linear and nonlinear systems. The theoretical background of autoregressive models [16,32] assumes relation

$$x(n) + a(1)\, x(n-1) + \cdots + a(na)\, x(n-na) = e(n)$$

allowing estimation of reliability limits

$$x(n) \in \hat{x}(n) \pm u_{\varepsilon/2} \sqrt{(1 + \sum_{j=1}^{m-1} h(j)^2)}\; v_e \tag{8}$$

for a chosen critical value $\varepsilon/2$, variance v_e and length m. Fig. 8 presents results of gas consumption prediction.

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Figure 8. Gas consumption prediction and its reliability limits

Non-linear systems used for signal prediction use in many cases artificial neural networks [8,4] and Widrow methods of adaptive signal processing. While linear autoregressive models are sufficient in many applications including both serially and seasonally related time series it is necessary to use non-linear systems in special cases. The basic neural network structure [8,24,23] presented in Fig. 9

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Figure 9. Basic structure of artificial neural networks

assumes its two layers of sizes $R - S1 - 1$ with a transfer function $F1$ including both sigmoidal and wavelet functions. Output of such a model for any input vector $\mathbf{P}_{R,1}$ is defined by relation

$$\mathbf{Y} = \mathbf{W}2 * F1\,(\mathbf{W}1 * \mathbf{P} + \mathbf{B}1) + \mathbf{B}2 - 1mm \tag{9}$$

where matrices $\mathbf{W}1_{S1,R}, \mathbf{W}2_{1,S1}$ define neural network coefficients and vectors $\mathbf{B}1_{S1,1}, \mathbf{B}2_{1,1}$ represent biases.

Gas consumption prediction assuming the specification of learning and validation parts is given in Fig. 10.

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Figure 10. Artificial neural network application to gas consumption prediction

Coefficients estimation assumes application of various optimization methods including genetic algorithms and gradient methods presented in Fig. 11.

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Figure 11. Comparison of gradient method and genetic search

4. CONCLUSION

The paper presents selected aspects of relation between information engineering and methods of signal and image processing. In this context several general algorithms of signal analysis and processing are presented as well together with application to biomedical and environmental signal processing. The basic goal of the paper is to show that the mathematical background is similar in various research areas and to point to general tools of information engineering and discrete data processing. Further information are available from the DSP research group address *http://phobos.vscht.cz*.

It seems that classical methods of signal processing developed in the 20th century will form basis for evolution of modern statistical digital signal processing methods in the 21st century. According to opinion of Prof. Simon Haykin [9] it will bring together mathematics and physics reconciling the ever-present tension between them allowing to (i) test the performance of algorithms with real-life data and (ii) learn from the data.

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Table 1. Testovací tabulka českých znaků

Testing	ěščřž	ýáíé	úůóň	ťď
table	ĚŠČŘŽ	ÝÁÍÉ	ÚŮÓŇ	ŤĎ
	escrz	yaie	uuon	td